CONGRUENT ZETA FUNCTIONS. NO.3

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3.1. **Definition of congruent Zeta function.**

DEFINITION 3.1. Let q be a power of a prime. Let $V = \{f_1, f_2, \dots, f_m\}$ be a set of polynomial equations in n-variables over $\mathbb{F}_q$. We denote by $V(\mathbb{F}_{q^s})$ the set of solutions of V in $(\mathbb{F}_{q^s})^n$. That means,

$$V(\mathbb{F}_{q^s}) = \{x \in (\mathbb{F}_{q^s})^n; f_1(x) = 0, f_2(x) = 0, \dots, f_m(x) = 0\}.$$

Then we define

$$Z(V/\mathbb{F}_q, T) = \exp(\sum_{s=1}^{\infty}(\frac{1}{s}\#V(\mathbb{F}_{q^s})T^s)).$$

EXERCISE 3.1. Compute congruent zeta function for $V = \{XY\}$ (an equation on two variables).

EXERCISE 3.2. Compute congruent zeta function for $V = \{X^2 + Y^2 - 1\}$ (an equation on two variables).