CATEGORIES, ABELIAN CATEGORIES AND COHOMOLOGIES.

YOSHIFUMI TSUCHIMOTO

Examples of categories

(1) (Sets)=(Category of all sets.) For $X, Y \in$ (Sets), the set of morphism $\mathrm{Hom}_{(\mathrm{set})}(X, Y)$ is defined to be the set of all maps from X to Y.
(2) (Groups)=(Category of all groups.) For $X, Y \in$ (Groups), the set of morphism $\mathrm{Hom}_{(\mathrm{group})}(X, Y)$ is defined to be the set of all group homomorphisms from X to Y.
(3) (Abelian groups), (Commutative Rings), are defined in a similar manner.
(4) For a given ring R, we define (R-modules) to be the category of all R-modules. Morphisms are R-module homomorphisms.
(5) (Top)=(Category of all topological spaces.) For $X, Y \in$ (Top), the set of morphism $\mathrm{Hom}_{(\mathrm{top})}(X, Y)$ is defined to be the set of all continuous maps from X to Y.
(6) (Hausdorff Sp.)=(the category of all Hausdorff spaces), (Compact Sp)=(the category of all Compact spaces) are defined in a similar manner.

Definition 2.1. A (covariant) **functor** F from a category $\mathcal{C}$ to a category $\mathcal{D}$ consists of the following data:

(1) An function which assigns to each object C of $\mathcal{C}$ an object $F(C)$ of $\mathcal{D}$.
(2) An function which assigns to each morphism f of $\mathcal{C}$ an morphism $F(f)$ of $\mathcal{D}$.

The data must satisfy the following axioms:

(functor-1) $F(1_C) = 1_{F(C)}$ for any object C of $\mathcal{C}$.
(functor-2) $F(f \circ g) = F(f) \circ F(g)$ for any composable morphisms f, g of $\mathcal{C}$.

By employing an axiom

(functor-2′) $F(f \circ g) = F(g) \circ F(f)$ for any composable morphisms

instead of axiom (functor-2) above, we obtain a definition of a **contravariant functor**.