COMMUTATIVE ALGEBRA

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11. regular element

DEFINITION 11.1. Let A be a commutative ring. Let M be an A-module. An element $x \in A$ is said to be M**-regular** if

$$0 \to M \overset{x\times}{\to} M$$

is exact.

PROPOSITION 11.2. *Let A be a ring, M, N two A-modules, and $x \in A$. Suppose that x is both A-regular and M-regular, and that $xN = 0$. Set $B = A/xA$ and $\bar{M} = M/xM$. Then:*

(1) $\mathrm{Tor}_n^A(M, B) = 0$ *for all* $n > 0$.
(2) $\mathrm{Ext}_A^n(M, N) \cong \mathrm{Ext}_B^n(\bar{M}, N)$ *for all* $n \geq 0$.
(3) $\mathrm{Tor}_n^A(M, N) \cong \mathrm{Tor}_n^B(\bar{M}, N)$ *for all* $n \geq 0$.