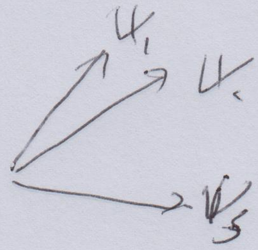
$\underset{\sqcup}{V}$：計量ベクトル空間、

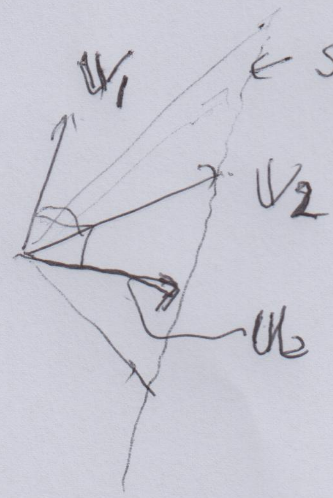


v_1, v_2, v_3

$$(v_i \cdot v_j)_{ij} = A$$

$$(a_1 v_1 + a_2 v_2 + a_3 v_3) \cdot (b_1 v_1 + b_2 v_2 + b_3 v_3)$$

$$= (a_1 \ a_2 \ a_3) A \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$



$\boldsymbol{v}_2 \cdot \boldsymbol{u}_3 = 0$

$$\Leftrightarrow (0\ 1\ 0) A \begin{pmatrix} t \\ u \\ 1 \end{pmatrix} = 0$$

$$(1\ 0\ 0) A \begin{pmatrix} s \\ 1 \\ 0 \end{pmatrix} = 0$$

$$(1\ 0\ 0) A \begin{pmatrix} t \\ u \\ 1 \end{pmatrix} = 0$$

行列の結合

$$(1\ 0\ 0) A \left(\boxed{\begin{matrix} s \\ 1 \\ 0 \end{matrix}} \boxed{\begin{matrix} t \\ u \\ 1 \end{matrix}} \right) = (0\ 0)$$

$$(0\ 1\ 0) A \begin{pmatrix} t \\ u \\ 1 \end{pmatrix} = 0$$

$$(1\ 0\ 0) A \begin{pmatrix} 1 & s & t \\ 0 & 1 & u \\ 0 & 0 & 1 \end{pmatrix}$$

$$= (* \ 0\ 0)$$

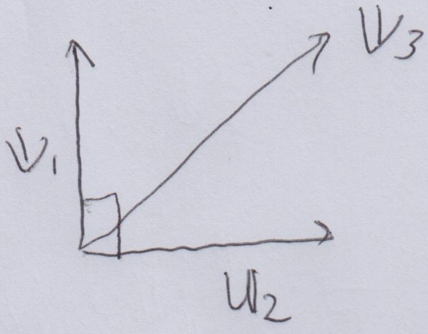
$$(1\ 0\ 0) A \begin{pmatrix} 1 & s & t \\ 0 & 1 & u \\ 0 & 0 & 1 \end{pmatrix} = (* \ 0\ 0)$$

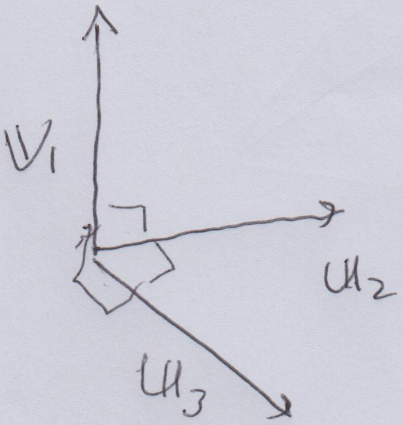
$$(0\ 1\ 0) A \begin{pmatrix} t \\ u \\ 1 \end{pmatrix} = 0$$

Q

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} A \begin{pmatrix} 1 & s & t \\ 0 & 1 & u \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} * & 0 & 0 \\ 0 & 1 & * \end{pmatrix}$$

内積を
Aを用いて
書ける.





$$v_1 \cdot \underset{(v_2 + s u_1)}{\overset{}{u_2}} = 0$$

$$(1\ \ 0\ \ 0)\, A \begin{pmatrix} s \\ 1 \\ 0 \end{pmatrix} = 0$$

$$u_3 = t u_1 + u v_2 + v_3 \rightarrow \begin{pmatrix} t \\ u \\ 1 \end{pmatrix}$$

$$v_1 \cdot u_3 = 0$$

$$(1\ \ 0\ \ 0)\, A \begin{pmatrix} t \\ u \\ 1 \end{pmatrix} = 0$$

$$\underline{u_2 \cdot u_3 = 0}$$
$$\Updownarrow$$
$$v_2 \cdot u_3 = 0$$

$$\begin{cases} v_1 \cdot u_3 = 0 \\ v_2 \cdot u_3 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} u_1 \cdot u_3 = 0 \\ u_2 \cdot u_3 = 0 \end{cases}$$

u_1, u_2, u_3 が直交してい

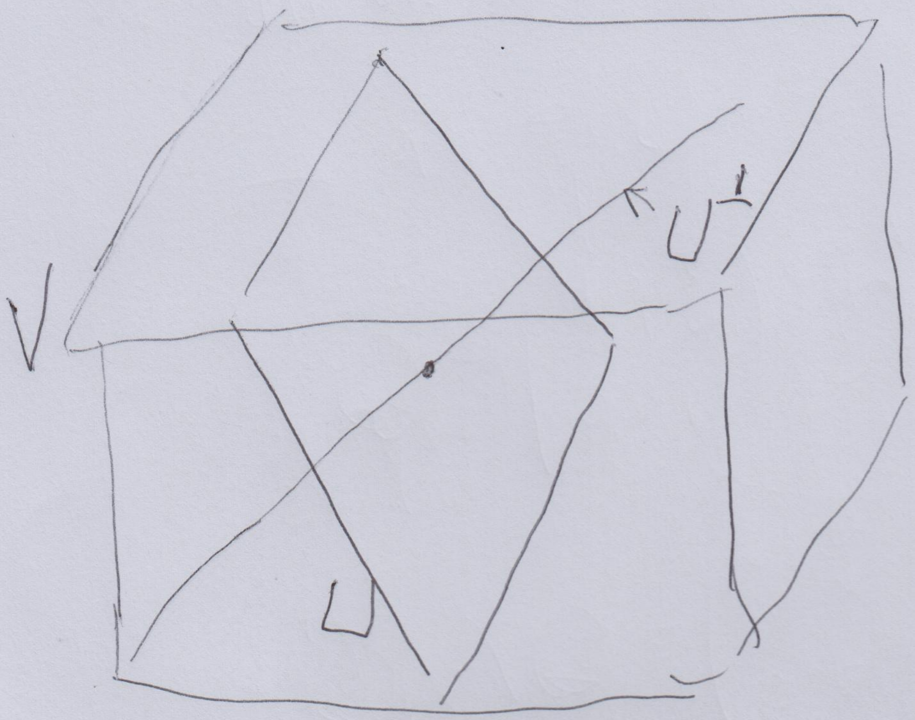
→ Aで書くと.

$$Q = \begin{pmatrix} 1 & s & t \\ 0 & 1 & u \\ 0 & 0 & 1 \end{pmatrix}$$ を用いて ⇐ vをuに変換

$${}^tQAQ = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

直交

内積をとりかえたあとの行列


V
U^{-1}
U

4.2の証明.

(1) $x, y \in U^{\perp}$ とする.

$((\lambda \cdot x) \in U^{\perp}?)$

$\lambda \in \mathbb{R}$ とし

$\forall u \in U$ に対し

$((\lambda \cdot x) \cdot u) = \lambda \underbrace{(x \cdot u)}_{\overset{\shortparallel}{0}} = \lambda \cdot 0 = 0.$ $\leftarrow x \in U^{\perp}$ $\therefore \lambda x \in U^{\perp}$

$(x+y) \cdot u = \underbrace{(x \cdot u)}_{\overset{\shortparallel}{0}} + \underbrace{(y \cdot u)}_{\overset{\shortparallel}{0}} = 0$

ゆえに $x + y \in U^{\perp}$

(3) $(U^{\perp})^{\perp} \supset U$

(4)

(2) $(\mathbb{v}+\mathbb{u}_0)$, $(\mathbb{v}+\mathbb{u}_1)$: 長さ最小とする

$$\mathbb{u}_1 = \mathbb{u}_0 + \mathbb{x} \quad (\mathbb{x} = \mathbb{u}_1 - \mathbb{u}_0) \in U$$

$$\|\mathbb{v}+\mathbb{u}_0+\mathbb{x}\|^2 = \|\mathbb{v}+\mathbb{u}_0\|^2$$

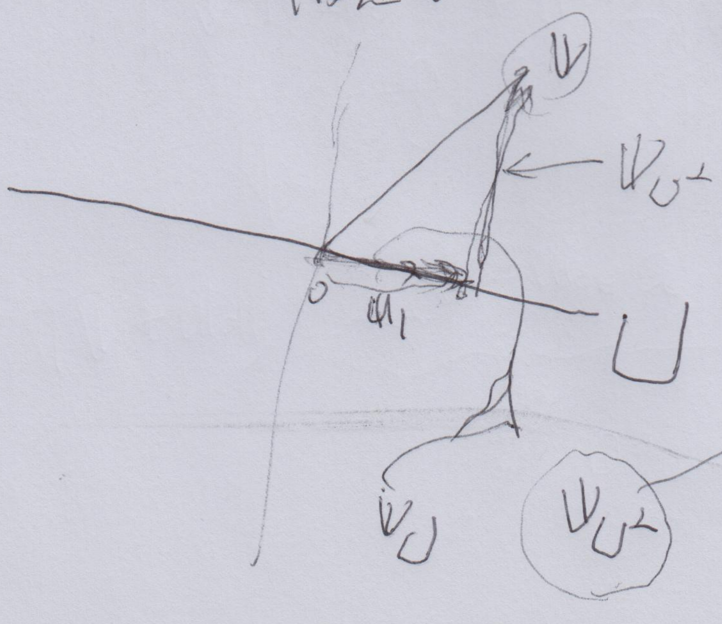
$$= \|\mathbb{v}+\mathbb{u}_0\|^2 + \underbrace{2((\mathbb{v}+\mathbb{u}_0)\cdot\mathbb{x})}_{(1)\text{により}0} + \|\mathbb{x}\|^2$$

$\|\mathbb{x}\|^2 = 0$ $\therefore \mathbb{x} = \mathbb{0}$

(3) ガチ

(4) $\boxed{\mathbb{u}_1 \neq 0}$ 仮定しといて下さい

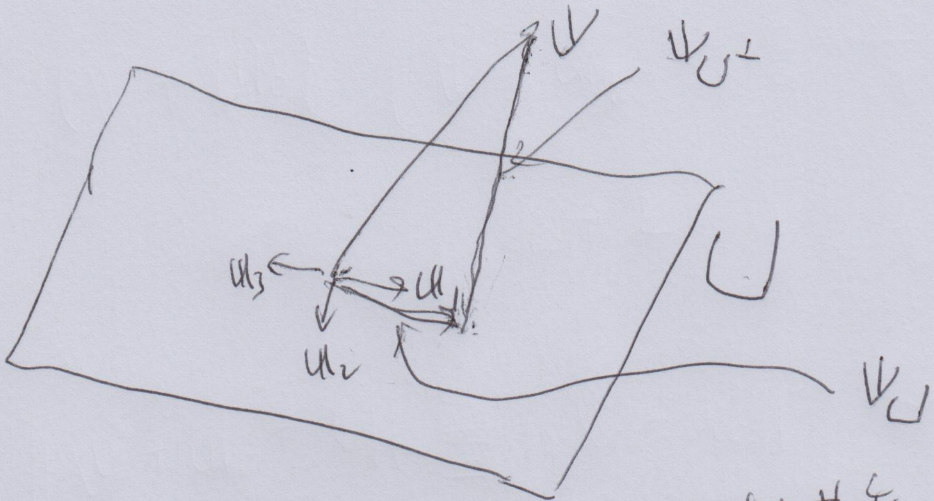
$$\|\mathbb{v}+s\mathbb{u}_1\|^2 = \|\mathbb{v}\|^2 + 2s(\mathbb{v}\cdot\mathbb{u}_1) + s^2\|\mathbb{u}_1\|^2$$



: sについて最小となるsをs_0とおくこと

$\boxed{\mathbb{v}+s_0\mathbb{u}_1}$ 答

s_0求めろ ガチ



$u_1, u_2, \cdots, u_n$: Wの正規直交基底

$\| v + c_1 u_1 + \cdots + c_n u_n \|$: 最小になる

(1)を使う

⊖ ~~uを $c_1 u_1 + \cdots + c_n u_n$ とす~~

→ $c_1, \cdots, c_n$ が求まる.

($v \cdot u_1, \cdots, v \cdot u_n$ を用いる)