AFFINGE GROUP SCHEMES 07

Let $\mathbb{k}$ be a ring. Let A be a $\mathbb{k}$-algebra. Let M be an A-module. A k-linear map

$$D : A \to M$$

is called a derivation if

$$D(xy) = D(x)y + xD(y) \qquad (\forall x, \forall y \in A).$$

Let A be a Hopf-algebra. A linear map $T : A \to A$ is called left-invariant if

$$\Delta T = (\mathrm{id} \otimes T) \circ \Delta.$$

The Lie algebra of a group G represented by A is the $\mathbb{k}$-vector space of left invariant $\mathbb{k}$-derivations $D : A \to A$.

$$\mathrm{Lie}(G) = \{D : A \to A;\ \text{left invariant derivation}\}$$

PROPOSITION 0.1. $\mathrm{Lie}(G)$ *is a Lie algebra. That means, it is a* $\mathbb{k}$-*vector space closed under commutotors.*

PROPOSITION 0.2. *There is a canonical bijection between* $\mathrm{Lie}(G(\mathbb{k}))$ *and* $G(\mathbb{k}[\epsilon]/(\epsilon^2))$.

EXAMPLE 0.3. (1) $\mathrm{Lie}(\mathrm{GL}_n(\mathbb{k}))=\mathfrak{gl}_n(\mathbb{k}) = M_n(\mathbb{k})$
(2) $\mathrm{Lie}(\mathrm{SL}_n(\mathbb{k}))=\mathfrak{sl}_n(\mathbb{k}) = \{A \in M_n(\mathbb{k}); \mathrm{tr}(A) = 0\}$